Transformations

A function is a rule that assigns exactly one output to each input. A transformation is a function that describes a change in the position, size or shape of a figure.

Rigid transformations only change in position. They are translations (slides), rotations (spins) and reflections (flips).

The input of a transformation is the preimage and the output is the image. The “change” is denoted by an arrow from one set of ordered pairs to the other.

$(x , y) \longrightarrow (x+1, y+4)$

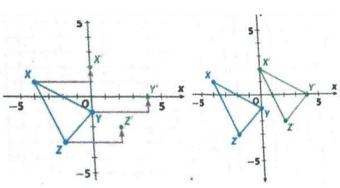
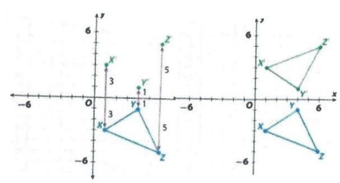
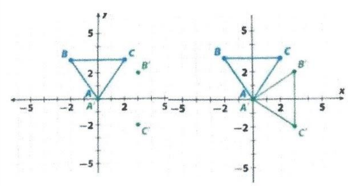
This transformation in particular is a translation of the preimage 1 unit right and 4 units up in the coordinate plane.

Line AB ⟶ Line A’B’

This line is now called A prime B prime when you say it out loud.

When you transform a preimage, the name of the preimages’ vertices change. They become called “primes.”

Rigid Transformations

Translations	Reflections	Rotations
Verbal description: Slides a figure along a straight line	**Verbal description:** Flips a figure across a line called the ***line of reflection***. Each point and its image are the same distance from this line.	**Verbal description:** Turns a figure around a given point called the ***center of rotation***. Image has the same size and shape as the preimage.
Visual description: 	**Visual description:** 	**Visual description:** 
Algebraic description: The figure slid 4 units right and 1 unit up. You would add 4 to the x-coordinate value and add 1 to the y-coordinate value. **Left/Right → Subt/Add x-coord** **Up/Down → Add/Subt y-coord**	**Algebraic description:** The figure flipped across the x-axis. I would highlight the line of reflection, in this case the x-axis. **Across the x-axis → take the opposite sign of the y-coord (x,-y)** **Across the y-axis → take the opposite sign of the x-coord (-x,y)** **Across y=x → switch locations (y,x)**	**Algebraic description:** The figure is rotated about the origin 90° clockwise. I would make a heavy dot on the center of rotation. **90° clockwise → switch signs of x-coord then switch locations (y,-x)** **90° CCW → switch signs of y-coord then switch locations (-y,x)** **180° → switch signs of each (-x,-y)** ***270° CW → same as 90° CCW*** ***270° CCW → same as 90° CW***

Algebraic representations of transformations

This section covers another way to view transformations for those of you that are more comfortable with algebra and haven't quite embraced the picturesqueness of geometry.

You can perform transformations from just a figure's coordinates. No graphing required. Sometimes the problem will be given to you this way, sometimes it will be given to you as just a graph with only the vertices labeled.

	X-coordinate rules	Y-coordinate rules
Add to the coordinate +	Move right →	Move up ↑
Subtract from the coordinate −	Move left ←	Move down ↓

Algebraic "Rules"

We can follow the given algebraic rules (what the problem tells us to do to the x-coordinate and the y-coordinate) to find the coordinates of the endpoints of the image without graphing.

Example #1

Triangle *LMN* with *L*(-1, 3), *M*(2, 6), and *N*(5, 3); (x, y) → (x + 3, y - 4)

L(-1 ***+ 3***, 3 ***- 4***), *M*(2 ***+ 3***, 6 ***- 4***), *N*(5 ***+ 3***, 3 ***- 4***)

L'(-2, -1), M'(5, 2), N'(8, -1) Your solution.

Example #2

Parallelogram *RSTU* with *R*(-1, 8), *S*(3, 10), *T*(3, 8), *U*(-1, 6); (x, y) → (x - 3, y - 6)

R(-1 ***- 3***, 8 ***- 6***), *S*(3 ***- 3***, 10 ***- 6***), *T*(3 ***- 3***, 8 ***- 6***), *U*(-1 ***- 3***, 6 ***- 6***)

R'(-4, 2), S'(0, 4), T'(0, 2), U'(-4, 0)

Example #3

Trapezoid *EFGH* with *E*(3, 4), *F*(7, 3), *G*(7, 0), *H*(-1, 2); (x, y) → (x - 6, y + 2)

E(3 ***- 6***, 4 ***+ 2***), *F*(7 ***- 6***, 3 ***+ 2***), *G*(7 ***- 6***, 0 ***+ 2***), *H*(-1 ***- 6***, 2 ***+ 2***)

E'(-3, 6), F'(1, 5), G'(1, 2), H'(-7, 4)